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Newton's Discovery of Gravity

How did he come to develop the concept that marked the beginning of modern science? In essence he did so by repetitively comparing the real world with a simplified mathematical representation of it

by I. Bernard Cohen

The high point of the Scientific Revolution was Isaac Newton's discovery of the law of universal gravitation: All objects attract each other with a force directly proportional to the product of their masses and inversely proportional to the square of their separation. By subsuming under a single mathematical law the chief physical phenomena of the observable universe Newton demonstrated that terrestrial physics and celestial physics are one and the same. In one stroke the concept of universal gravitation revealed the physical significance of Johannes Kepler's three laws of planetary motion, solved the thorny problem of the origin of the tides and accounted for Galileo Galilei's curious and unexplained observation that the descent of a free-falling object is independent of its weight. Newton had achieved Kepler's goal of developing a physics based on causes.

The momentous discovery of universal gravitation, which became the paradigm of successful science, was not the result of an isolated flash of genius; it was the culmination of a series of exercises in problem solving. It was a product not of induction but of logical deductions and transformations of existing ideas. The discovery of universal gravity brings out what I believe is a fundamental characteristic of all great breakthroughs in science from the simplest innovations to the most dramatic revolutions: the creation of something new by the transformation of existing notions.

Newton developed the concept of universal gravity in the first few months of 1685, when he was 42. Physicists have usually made their greatest contributions at a much earlier age, but Newton was still in what he called "the prime years of my life for invention." The documents that have enabled me to date the discovery also make it possible to reconstruct the process that led to it.

A decisive step on the path to universal gravity came in late 1679 and early 1680, when Robert Hooke introduced Newton to a new way of analyzing motion along a curved trajectory. Hooke had cleverly seen that the motion of an orbiting body has two components, an inertial component and a centripetal, or center-seeking, one. The inertial component tends to propel the body in a straight line tangent to the curved path, whereas the centripetal component continuously draws the body away from the inertial straight-line trajectory. In a stable orbit such as that of the moon the two components are matched, so that the moon neither veers away on a tangential path nor spirals toward the earth.

The concept of a centripetal force replaced the older and misleading notion of a centrifugal, or center-fleeing, force. René Descartes and Christiaan Huygens had analyzed curved motion in terms of such a centrifugal force. Descartes, for example, had investigated the movement of a ball on the inner surface of a hollow cylinder and the movement of water in a bucket swung in a circle. The ball and the water seemed to flee the center of the system, and so Descartes attributed their motion to the influence of a centrifugal force. It is now clear there is no such force; a center-fleeing force cannot be traced to the interaction of physical objects. The illusion of a centrifugal force comes about when a moving object is viewed from a rotating frame of reference.

With the change in outlook from centrifugal to centripetal force came an appreciation of the fundamental role of the central body. The centrifugal analysis had focused on the revolving object, whose "endeavor to recede" from the center seems to be independent of the properties of the central body. The concept of a centripetal force, in contrast, depends fundamentally on the central body, toward which the revolving object is impelled or attracted. The interaction of the central, attracting body with the revolving, attracted object can obviously be expected to have a part in any theory of gravitation.

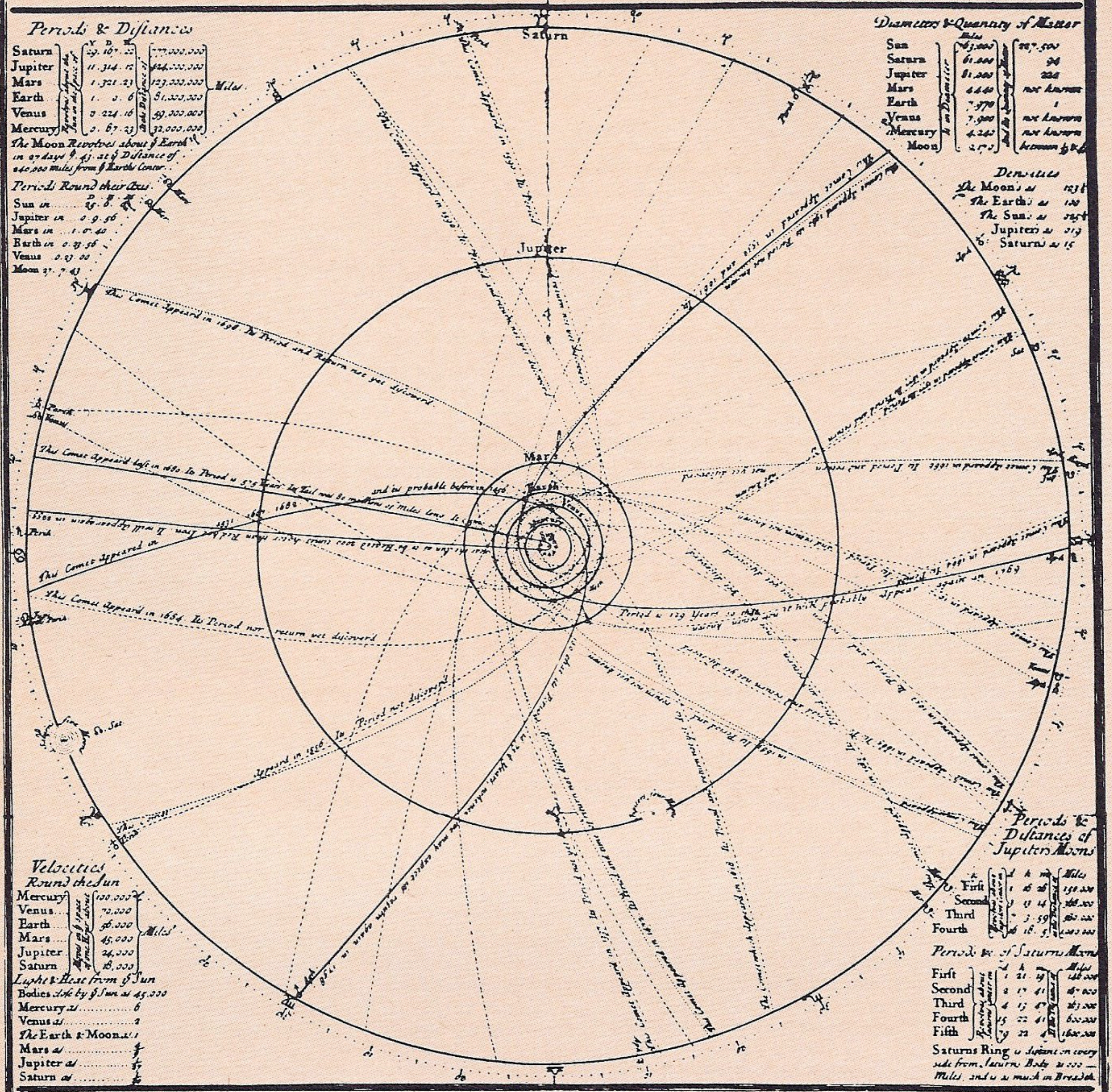
Hooke's analysis of curved motion may seem to be such an obvious and immediate consequence of the Cartesian principle of inertia that Newton would not have needed Hooke to instruct him on the subject as late as 1679. Newton had more or less accepted the inertial principle some 20 years earlier. Nevertheless, Newton, like Descartes and Huygens, was so mired in the concept of centrifugal endeavor that the full implications of inertial physics were far from obvious to him.

On November 24, 1679, Hooke wrote to Newton suggesting that they engage in a private "philosophical" correspondence on scientific topics of mutual interest. Six years earlier they had clashed publicly over Newton's experiments and theories on the prismatic dispersion of light and on the nature of color. Hooke was only one of several investigators who had rejected Newton's optical theories. Newton was so vexed at having to defend his work that he vowed to abandon "philosophy" (physical science) because she was "so litigious a lady" that a man who had anything to do with her would have to spend the rest of his life defending his opinions.

Hooke had since become secretary of the Royal Society of London. In spite of the earlier controversy his letter to Newton was friendly and gracious. The letter invited Newton to comment on any of Hooke's hypotheses or opinions, particularly on the notion of "compounding the celestiall motions of the planetts [out] of a direct motion by the tangent & an attractive motion towards the central body." This sentence was apparently Newton's introduction to the idea of decomposing curved motion into an inertial component and a centripetal one. There is no evidence that he had yet reached Hooke's level of understanding of circular motion. Indeed, Newton still often spoke of orbital motion in terms of centrifugal force.

In his letter Hooke ventured the suggestion that the centripetal force drawing a planet toward the sun varies inversely as the square of the separation. At this point Hooke was stuck. He could not see the dynamical consequences of his own deep insight and therefore could not make the leap from intuitive hunch and guesswork to exact science. He

Mr. WHISTON'S SCHEME of the SOLAR SYSTEM EPITOMIS'D. To wch is annex'd A Translation of part of ye General *Scholium* at ye end of ye second Edition of Sr. *Isaac Newtons* Principia. *Concerning God.*



THIS most Elegant System of the Planets and Comets could not be produced but by and under the Contrivance and Dominion of an Intelligent and Powerful Being. And if the Fixed Stars are the Centers of such other Systems, all these being Framed by the like Council will be Subject to the Dominion of *One*, especially seeing the Light of the Fixed Stars is of the same Nature with that of the Sun, and the Light of all these Systems passes mutually from one to another. He governs all things, not as ye Soul of the World, but as the Lord of the Universe, and because of his Dominion he is wont to be called Lord God παντοκράτωρ i. e. Universal Emperor) for God is a Relative word, and has Relation to Servants: And the Deity is the Empire of God, not over his own Body (as is the Opinion of those, who make him the Soul of the World) but over his Servants. The *Supreme God* is a Being Eternal, Infinite Absolutely Perfect; but a Being however Perfect without Dominion, is not *Lord God*: For we say, *my God, your God, the God of Israel*, but we do not say, *my Eternal, your Eternal, the Eternal of Israel*, we do not say *my Infinite, your Infinite, ye Infinite of Israel*; we do not say *my Perfect, your Perfect, the Perfect of Israel*. These Titles have no Relation to Servants. The word God frequently signifies Lord, but every Lord is not God. The Empire of a Spiritual being constitutes God, true Empire constitutes True God, Supreme the Supreme, Feigned the Feigned. And from his true Empire it follows that the true God is Living, Intelligent & Powerful, from his other Perfections that he is the Supreme or Supremely Perfect. He is *Eternal & Infinite, Omnipotent* and *Omnipresent* that is, he endures from Eternity to Eternity, and he is present from Infinity to Infinity, he Governs all Things, and Knows all Things which are or which can be known. He is not Eternity or Infinity, but he is Eternal and Infinite, he is not Duration or Space, but he Endures and is Present. He endures always and is present everywhere and by existing always and everywhere, he Constitutes Duration and Space, Eternity and Infinity. Where as every Particle of Space is *always*, and every Individual Moment of Duration is *every where*, certainly the Framer and Lord of the Universe shall not be (*nunquam nusquam*) *never no where*. He is Omnipresent not Virtually only, but also Substantially, for Power without Substance cannot Subsist. In him are containd and moved all things (so the Antients thought *) but without mutual Passion God suffers nothing from the Motions of Bodies: Nor do they suffer any Resistance from the Omnipresence of God. It is confess'd that the Supreme God exists Necessarily, and by the same Necessity he is *always* and *every where*. Whence also he is wholy Similar, all Eye, all Ear, all Brain, all Arm, all the Power of Perceiving Understanding and Acting. But after a manner not at all Corporeal, after a manner not like that of Men, after a manner wholly to us unknown. As a Blind Man has no notion of Colours, so neither have we any notion of the manner how the most Wise God perceives and understands all things. He is wholly destitute of all Body and of all Bodily shape, and therefore cannot be seen, heard, nor touched, nor ought to be Worshiped under the Representation of any thing Corporeal. We have Ideas of his Attributes, but we know not at all what is the Substance of any thing whatever. We see only the Figures and Colours of Bodies, we hear only Sounds, we touch only the outward Surfaces, we smell only Odours and tast Tasts, but we know not by any sence or reflex Act the inward Substances, and much less have we any Notion of the Substance of God: We know him only by his Properties and Attributes and by the most Wise and Excellent Structure of things, and by Final Causes, but we Adore and Worship him on account of his Dominion. For God, without Dominion, Providence & Final Causes is nothing else but Fate and Nature

* [illegible]

Engrav'd and Sold by John Senex at the Globe in Salisbury Court near Fleetstreet: Where may be had Dr Halley's Scheme of the Total Eclipse of the Sun which will be in 1724. Also his Zodiack containing all the Stars in the Way of the Moon and Planets. Useful in Astronomical Observations and for finding the Longitude at Sea

could go no further because he lacked both the mathematical genius of Newton and an appreciation of Kepler's law of areas, which figured prominently in Newton's subsequent approach to celestial dynamics. The law of areas states that the radius vector from the sun to a planet sweeps out equal areas in equal times.

On November 28 Newton wrote to Hooke that before reading Hooke's letter of the 24th he did not "so much as heare (that I remember) of your Hypotheses of compounding the celestial motions of the Planets of a direct motion by the tangent to the curve" and an "attractive" motion toward the sun. Having admitted that Hooke's analysis was new to him, Newton immediately changed the subject to a fancy of his own: the effect of the earth's rotation on a free-falling object. If a dropped object could pass through the rotating earth, what path would the object take? Newton had incorrectly concluded that it would follow a spiral trajectory.

In Hooke's next letter, dated December 9, he caught Newton's error and pointed out that the path "would resemble an Elleipse." Hooke was eager to get Newton going on the problem of planetary motion, and so he suggested that the correct description of an object falling through the earth and his own analysis of planetary motion were both cases of "Circular motions compounded by a Direct motion and an attractive one to a center."

On December 13, 1679, Newton responded guardedly to Hooke's correction but did not comment on his proposed analysis of circular motion. Hooke did not give up. In a letter written on January 6, 1680, he returned to his thesis about curved motion and repeated the quantitative supposition that the centripetal attraction is inversely proportional to the square of the distance. From this supposition Hooke concluded that the velocity of the revolving body is inversely proportional to the distance from the center. He then pointed out that his analysis "doth very Intelligibly and truly make out all the Appearances of the Heavens." Newton did not reply.

On January 17 Hooke sent a short supplementary letter in which he wrote: "It now remaines to know the proprietys of a curve Line (not circular nor concentricall) made by a centrall attractive power which makes the velocitys of Descent from the tangent Line or equall straight motion at all Distances in a Duplicate proportion reciprocally taken." In modern terminology Hooke's problem can be paraphrased as follows: If a central attractive force causes an object to fall away from its inertial path and move in a curve, what kind of curve results if the attractive force varies inversely as the square of the distance? He concluded: "I doubt not but that by your excellent method you will easily find out what that Curve must be, and its proprietys, and suggest a physicall Reason of this proportion."

Newton evidently did do almost that. He proved that an ellipse would satisfy the conditions outlined by Hooke. Nevertheless, he did not communicate the result of this proof to Hooke or to anyone else until August, 1684, when he was visited by Edmund Halley, the astronomer and mathematician. Halley came to see Newton in order to ask "what he thought the Curve would be that would be described by the Planets, supposing the force of attraction towards the Sun to be reciprocal to the square of their distance from it." The problem had been much discussed by the Royal Society. Halley and Christopher Wren were unable to solve it, and Hooke never produced a solution, although he maintained he had found one.

When Newton heard the question, he responded immediately: an ellipse. Halley asked him how he knew and Newton replied: "I have calculated it." Newton apparently could not find the calculations, but at Halley's urging he wrote them up for the Royal Society in the small tract *De Motu* (*Concerning Motion*). In *De Motu* Newton described his work on terrestrial and celestial dynamics, including his ideas on motion in free space and in a resistive medium. Newton must have finished *De Motu* by December 10, 1684, because Halley told the Royal Society then that Newton had recently shown him the curious treatise.

The exact progression of Newton's ideas in the time between his correspondence with Hooke and his completion of the first draft of *De Motu* is not documented. Nevertheless, I am certain it was Hooke's method of analyzing curved motion that set Newton on the right track. Although not all historians would agree with me, I believe the approach Newton takes to terrestrial and celestial dynamics in *De Motu*, which he further developed the following spring in the first book of the *Philosophiae Naturalis Principia Mathematica*, represents his thinking on planetary dynamics inspired by his correspondence with Hooke. In a few autobiographical manuscripts Newton said the correspondence either preceded or coincided with his demonstration published first in *De Motu* and then in the *Principia* that an object that has an inertial motion and is subject to an inverse-square centripetal force moves in an elliptical orbit.

It was this demonstration that brought out the physical significance of Kepler's law of elliptical orbits (the law stating that each planet moves in an elliptical path with the sun at one focus of the ellipse). The modern reader may be surprised that it was not Kepler but Newton who revealed the fundamental nature of Kepler's laws of planetary motion. Before the publication of the *Principia*, however, these laws (which were even called hypotheses) were not as highly respected as they came to be afterward.

Kepler's law of areas in particular had a diminished status in the 17th century. Most astronomical works did not even mention it. For example, Thomas Streete's *Astronomia Carolina*, from which Newton copied Kepler's third law (The cube of the average distance of a planet from the sun is proportional to the square of the orbital period), never discusses the law of areas or hints at its existence. Most 17th-century astronomers calculated planetary positions not by the law of areas but by a construction based on a uniformly rotating vector emanating from the empty focus of the planet's elliptical orbit [*see top illustration on page 174*]. Since astronomers rarely employed the law of areas, it required extraordinary perception for Newton to see its significance. Newton was the one who elevated Kepler's law of areas to the status it enjoys today.

The very first proposition of the *Principia* (and the discussion at the beginning of *De Motu*) develops the dynamical significance of the law of areas by proving that the curved motion described by the law is a consequence of centripetal force. The proof, which has three parts, shows how well Newton had learned Hooke's technique of decomposing curved motion into an inertial component and a centripetal one.

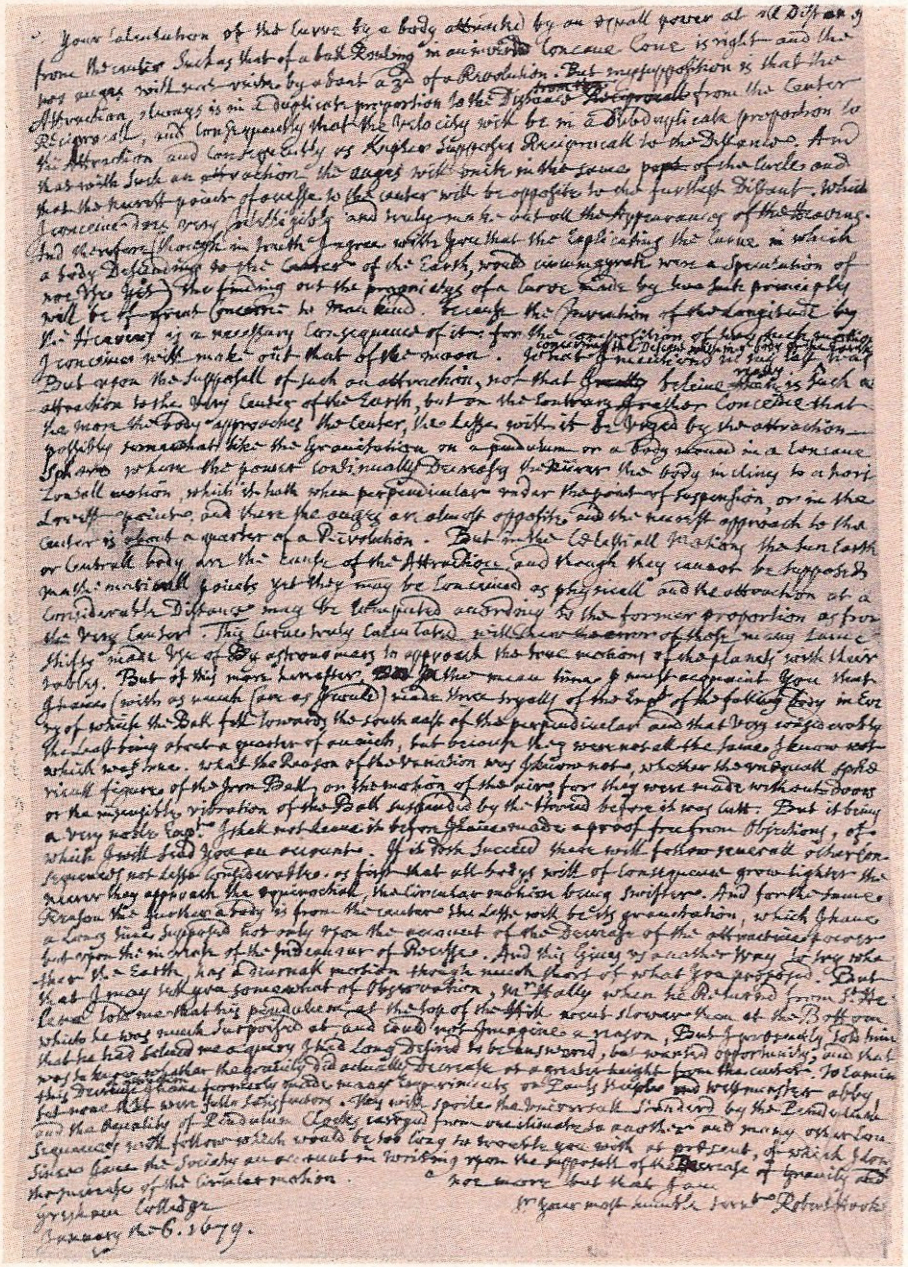
In the first part of the proof Newton considers a body moving along a straight line with a constant velocity. The line is divided into equal intervals to indicate that the body moves equal distances in equal times. A point P is chosen at a distance h above the line of motion. The triangles formed by connecting P to any of the equal intervals all have the same area because they have equal bases and the same altitude h. By this simple analysis Newton revealed

NEWTONIAN SYSTEM OF THE WORLD was diagrammed by William Whiston, who succeeded Newton as Lucasian Professor at the University of Cambridge. The diagram is from Whiston's broadside "Scheme of the Solar System Epitomis'd," published in 1724. The planets and the satellites of Jupiter and Saturn are shown orbiting the sun under the action of universal gravity. Remarkably, Whiston also included the orbits of comets. Newton had shown that orbits of comets are ellipses or parabolas in which a vector from the sun to the comet sweeps out equal areas in equal times. Below the diagram is Whiston's translation of part of the final General Scholium of the *Principia* (which is from the second edition, published in 1713). There Newton wrote that "This most Elegant System of the Planets and Comets could not be produced but by and under the Contrivance and Dominion of an Intelligent and Powerful Being."

an unexpected relation between inertial motion and the law of areas.

In the second part of the proof the body moves as before initially, but at the end of the second interval it receives an impulsive force—a blow—toward *P*. Therefore in the third interval the body no longer moves along the original straight line but rather along another straight line closer to *P*. Newton again showed by geometry that the triangle formed by connecting *P* to the ends of the trajectory traced in the second interval has the same area as the triangle formed by connecting *P* to the ends of the trajectory traced in the third interval.

In the third part the body is given a blow toward *P* at the end of each interval. As a result the body moves in a polygonal path around *P*. Again the area relation holds. In the limiting case where the interval between blows approaches zero the body is subject to a continuous force directed toward *P* and the polygonal path becomes a smooth curve or orbit. In this way Newton proved that a centripetal force generates a curve according to the law of areas.



LETTER TO NEWTON from Robert Hooke includes Hooke's views on the analysis of motion along a curved trajectory. (The letter is dated January 6, 1679, according to a version of the Julian calendrical system in which the year started in March; the modern calendrical system puts the date at January 6, 1680.) In the second sentence Hooke proposes that "the Attraction is always in a duplicate proportion to the Distance from the Center Reciprocall" (that is, the attraction is inversely proportional to the distance squared). As a result "the Velocity will be in a subduplicate proportion to the Attraction and consequently as Kepler supposes Reciprocall to the Distance." Hooke states that this analysis explains "all the Appearances of the Heavens." He stresses the importance of "finding out the proprietys" of curves because longitudes, which are "of great Concerne to Mankind," can be derived from the moon's curved motion.

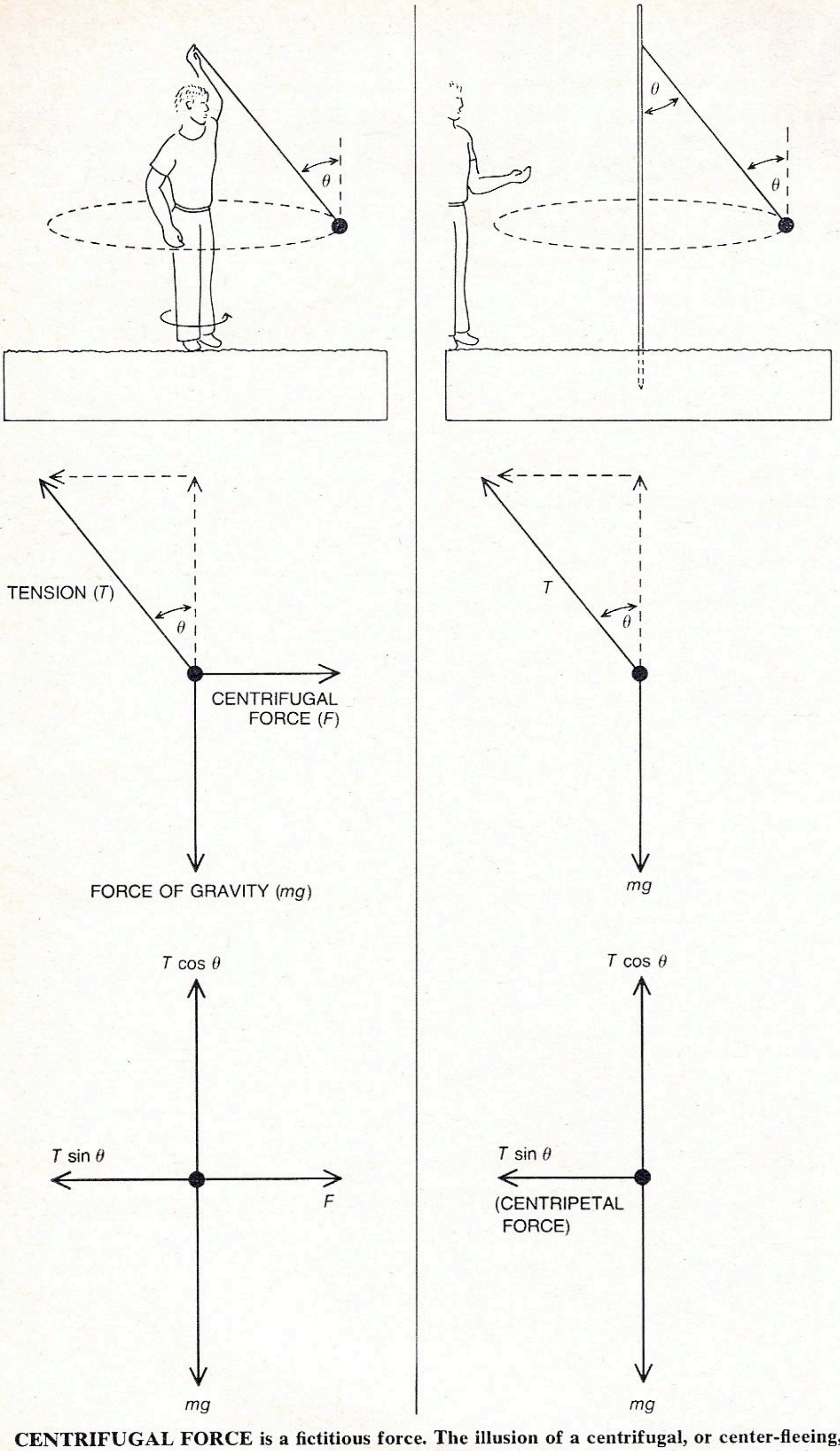
The second proposition of the *Principia* proves the converse: Motion in a curve described by the law of areas implies a centripetal force. With these two propositions Newton demonstrated that the law of areas is a necessary and sufficient condition for inertial motion in a central-force field.

The two propositions are part of a sequence of demonstrations that begins with the law of areas and ends with a proof that an elliptical orbit requires an inverse-square centripetal force. This sequence of demonstrations, presented both in the *Principia* and in *De Motu*, marks a profound discontinuity in the history of the exact sciences. The demonstrations introduced a radically new celestial dynamics based on new concepts of force, momentum, mass and inertia and a wholly novel quantitative measure of dynamical force. The subtitle of Kepler's *Astronomia Nova* set the goal of creating a "celestial physics based on causes." Newton achieved this goal, of which Kepler had had only a visionary glimpse. Neither Galileo nor Descartes had conceived of such a celestial dynamics. And the Newtonian formulation left even the great physicist Huygens far behind.

From the early draft of *De Motu* that Newton probably wrote in November, 1684, it is clear he had not yet developed the concept of universal gravitation. The draft discusses centripetal force directed toward the focus of an ellipse and concludes with the scholium, "Therefore the major planets revolve in ellipses having a focus in the center of the sun, and radii drawn [from the planets] to the sun describe areas proportional to the times, entirely as Kepler supposed. . . ."

Newton neither proved this scholium nor continued to believe it for long, and strictly speaking it is false. As he soon realized, the planets do not move according to the law of areas in simple Keplerian elliptical orbits with the sun at a focus. Instead the focus lies in the common center of mass because not only does the sun attract each planet but also each planet attracts the sun (and the planets attract one another). If Newton had already formulated his principle of universal gravitation, he would not have proposed the erroneous scholium.

Newton quickly realized he had not proved that the planets move precisely according to the law of elliptical orbits and the law of areas. He had only found that the laws hold for a one-body system: a single point mass moving with an initial component of inertial motion in a central-force field. He recognized that the one-body system corresponds not to the real world but to an artificial situation that is easier to investigate mathematically. The one-body system reduces the earth to a point mass and the sun to an immobile center of force.



CENTRIFUGAL FORCE is a fictitious force. The illusion of a centrifugal, or center-fleeing, force can arise when a moving object is viewed from a rotating frame of reference (*left*), as when a ball is swung at the end of a string by an observer who rotates with the same angular speed as the ball. Two known forces act on the ball: the tension of the string and the force of gravity. The ball is not accelerating in the vertical direction, and so all vertical forces acting on it must be in balance; in particular the vertical component of the tension cancels the force of gravity. Since the observer and the ball are rotating together, the ball appears to be at rest and it seems that the horizontal forces should also be in balance. As a result the observer postulates a centrifugal force that cancels the horizontal component of the tension. No such force, however, can be traced to the interaction of physical objects. A different analysis of forces results (*right*) when the ball is rotating in the same way but the observer is at rest. In this stationary frame of reference the observer sees the same vertical forces on the ball as he saw in the rotating frame. In the horizontal direction, however, the ball is not at rest with respect to the observer but is moving in a circle. In other words, the ball accelerates continuously toward the center, so that the horizontal forces should not be expected to balance. The ball is subject to a centripetal, or center-seeking, force which is the horizontal component of the tension of the string. The centripetal force can be traced to the interaction of two physical objects: the string and the ball.

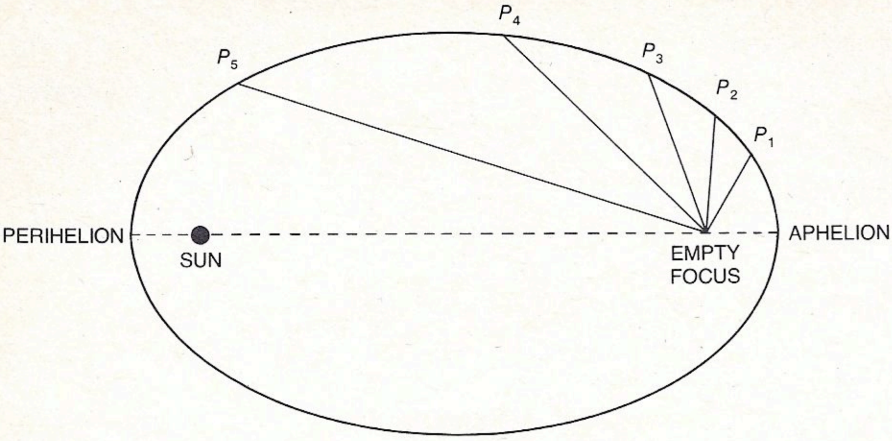
What enabled Newton to transcend the one-body system was his appreciation of the consequences of his third law of motion: the law of action and reaction. This law is perhaps the most original of his three laws of motion (the other two are the law of inertia and the force law). One testimonial to its novelty is that even today it is often employed incorrectly by those who relate it not to an impact situation or to the interaction of bodies but to a supposed condition of equilibrium.

The development of Newton's thinking on action and reaction after he completed the first draft of *De Motu* is set out in the opening sections of the first book of the *Principia*. In the introduction to the 11th section Newton explains that he has confined himself so far to a situation that "hardly exists in the real world," namely the "motions of bodies attracted toward an unmoving center." The situation is artificial because "attractions customarily are directed toward bodies and—by the third law of motion—the actions of attracting and attracted bodies are always mutual and equal." As a result, "if there are two bodies, neither the attracting nor the attracted body can be at rest." Rather, "both bodies (by the fourth corollary of the laws) revolve about a common center, as if by a mutual attraction."

Newton had seen that if the sun pulls on the earth, the earth must also pull on the sun with a force of equal magnitude. In this two-body system the earth does not move in a simple orbit around the sun. Instead the sun and the earth each move about their mutual center of gravity. A further consequence of the third law of motion is that each planet is a center of attractive force as well as an attracted body; it follows that a planet not only attracts and is attracted by the sun but also attracts and is attracted by each of the other planets. Here Newton has taken the momentous step from an interactive two-body system to an interactive many-body system.

In December, 1684, Newton completed a revised draft of *De Motu* that describes planetary motion in the context of an interactive many-body system. Unlike the earlier draft the revised one concludes that "the planets neither move exactly in ellipses nor revolve twice in the same orbit." This conclusion led Newton to the following result: "There are as many orbits to a planet as it has revolutions, as in the motion of the Moon, and the orbit of any one planet depends on the combined motion of all the planets, not to mention the actions of all these on each other." He then wrote: "To consider simultaneously all these causes of motion and to define these motions by exact laws allowing of convenient calculation exceeds, unless I am mistaken, the force of the entire human intellect."

There are no documents that indicate



PLANETARY POSITIONS were often found in the 17th century not by Kepler's law of areas but by a construction based on a uniformly rotating radius vector that emanates from the empty focus of a planet's elliptical orbit. The position of a planet (P_1, P_2, P_3, P_4, P_5) at successive moments is the intersection of the ellipse and the vector. Kepler's law of areas states that the radius vector from the sun to a planet sweeps out equal areas in equal times. As a result the planet moves slower at aphelion than at perihelion. The diagrammed construction gives the same result. Correction factors were added to make the construction fit the data more accurately.

how, in the month or so between writing the first draft of *De Motu* and revising it, Newton came to perceive that the planets act gravitationally on one another. Nevertheless, the passage cited above expresses this perception in unambiguous language: *"eorum omnium actiones in se invicem"* ("the actions of all these on each other"). A consequence of this mutual gravitational attraction is that all three of Kepler's laws are not strictly true in the world of physics but are true only for a mathematical construct in which point masses that do not interact with one another orbit either a mathematical center of force or a stationary attracting body. The distinction Newton draws between the realm of mathematics, in which Kepler's laws are truly laws, and the realm of physics, in which they are only "hypotheses," or approximations, is one of the revolutionary features of Newtonian celestial dynamics.

I have assumed that the third law of motion was the key factor in the reasoning that led Newton to suggest mutual gravitational perturbations of planetary orbits. There is no direct evidence for my assumption because no documents exist in which there is an antecedent version of his statement "the actions of all these on each other." Nevertheless, there is strong indirect evidence. In the spring of 1685, a few months after revising *De Motu,* Newton was well on his way to finishing the first draft of the *Principia.* In the initial version of what was to become a second book, "The System of the World," he spelled out the steps that led him to the concept of planetary gravitational interactions. In these steps the third law of motion has the chief role, and I see no reason to believe they are not the same steps that led him to the same concept a few months earlier when he revised *De Motu.*

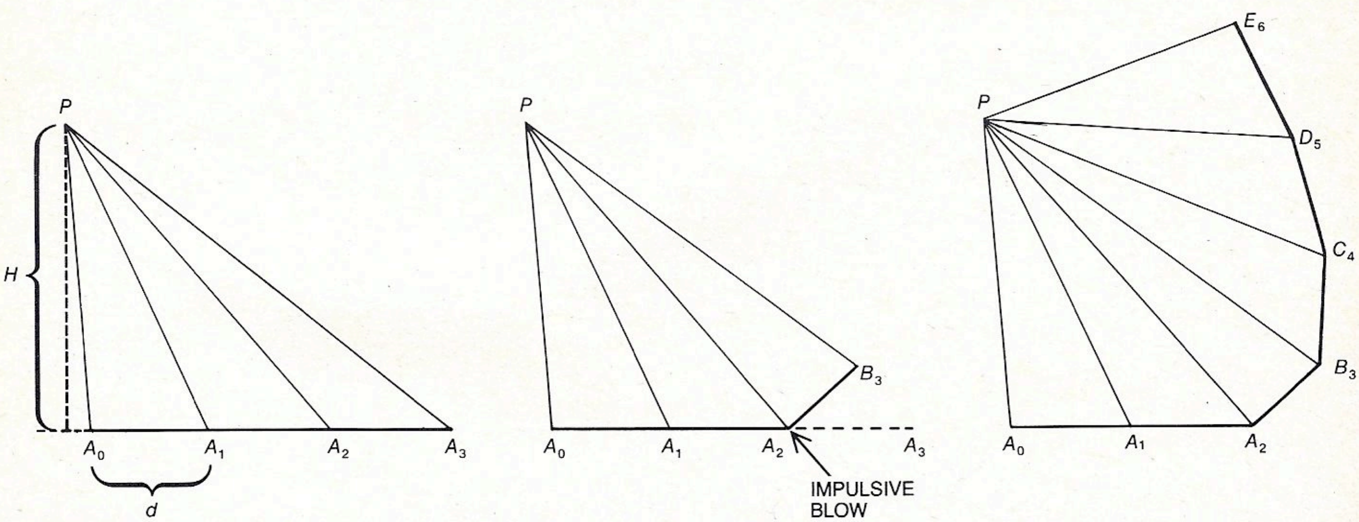
Here are two passages from the first draft of "The System of the World" (translated from the Latin by Anne Whitman and me) that bring out the crucial role of the third law of motion:

"20. *The agreement between the analogies.*

"And since the action of centripetal force upon the attracted body, at equal distances, is proportional to the matter in this body, it is reasonable, too, that it is also proportional to the matter in the attracting body. For the action is mutual, and causes the bodies by a mutual endeavor (by law 3) to approach each other, and accordingly it ought to be similar to itself in both bodies. One body can be considered as attracting and the other as attracted, but this distinction is more mathematical than natural. The attraction is really that of either of the two bodies toward the other, and thus is of the same kind in each of the bodies.

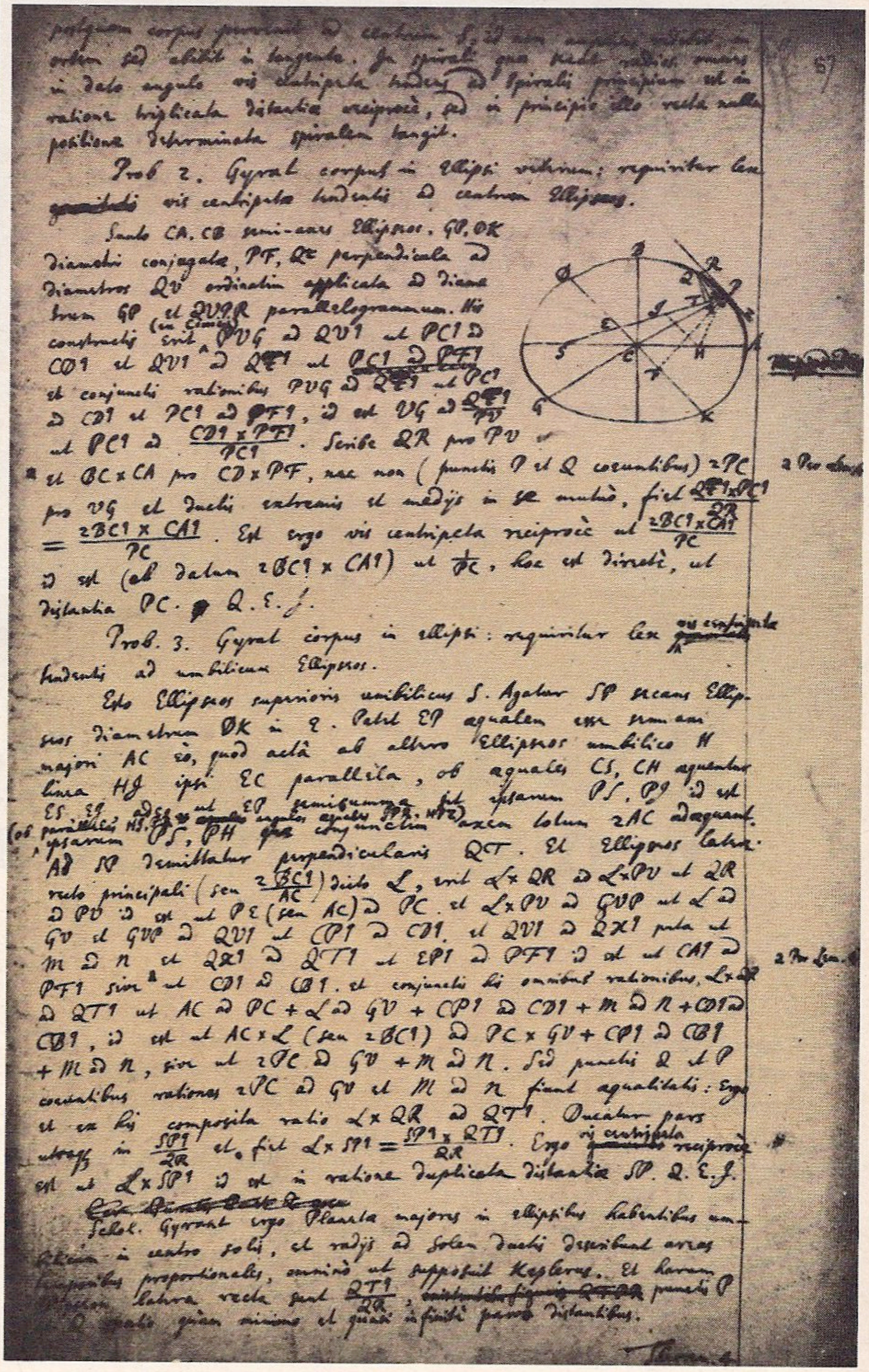
"21. *And their coincidence.*

"And hence it is that the attractive force is found in both bodies. The sun attracts Jupiter and the other planets,



CENTRIPETAL FORCE generates a curved trajectory consistent with the law of areas. This property of a centripetal force was demonstrated by Newton in the first proposition of the *Principia* and in the discussion at the beginning of the short tract *De Motu* (*Concerning Motion*). Newton began (*left*) by considering a body moving in a straight line at a constant speed. The body starts at A_0 and after successive equal intervals reaches first A_1, then A_2 and so on. A point P is chosen above the line of motion. The triangles A_0PA_1, A_1PA_2, A_2PA_3 and so forth all have the same area because they have equal bases and the same altitude. In a second stage of the analysis (*middle*) the body begins as before but at A_2 receives an impulsive blow toward P. Now the body moves along a straight line not to A_3 but to B_3. Newton showed by geometric methods that the triangles A_1PA_2 and A_2PB_3 have the same area. If the body receives a blow toward P at the end of each interval (*right*), it moves in a polygonal path around P. Again triangles can be formed that have the same area. In the limiting case where the time between blows approaches zero the body is subject to a continuous centripetal force directed toward P and the polygonal path becomes a smooth curve. Area is still conserved. This proof brought out the dynamical significance of the law of areas.

Jupiter attracts its satellites and similarly the satellites act on one another and on Jupiter, and all the planets on one another. And although the actions of each of a pair of planets on the other can be distinguished from each other and can be considered as two actions by which each attracts the other, yet inasmuch as they are between the same two bodies they are not two but a simple operation between two termini. Two bodies can be drawn to each other by the contraction of one rope between them. The cause of the action is twofold, namely the disposition of each of the two bodies; the action is likewise twofold, insofar as it is upon two bodies; but insofar as it is between two bodies it is



PAGE FROM A DRAFT OF "DE MOTU" that Newton probably wrote in November, 1684, is in his handwriting. In *De Motu* Newton discussed terrestrial and celestial dynamics, including the idea of centripetal force directed toward the focus of an ellipse. The page ends with the scholium, "Therefore the major planets revolve in ellipses having a focus in the center of the sun, and radii drawn [from the planets] to the sun describe areas proportional to the times, entirely as Kepler supposed. . . ." The scholium is false, and the nature of the error indicates that Newton had not yet developed the concept of universal gravitation. As Newton soon realized, the focus of the orbits of the planets is not the sun but the center of mass common to the planets and the sun. Not only does the sun attract each planet but also each planet attracts the sun.

single and one. There is not, for example, one operation by which the sun attracts Jupiter and another operation by which Jupiter attracts the sun, but one operation by which the sun and Jupiter endeavor to approach each other. By the action by which the sun attracts Jupiter, Jupiter and the sun endeavor to approach each other (by law 3), and by the action by which Jupiter attracts the sun, Jupiter and the sun also endeavor to approach each other. Moreover, the sun is not attracted by a twofold action toward Jupiter, nor is Jupiter attracted by a twofold action toward the sun, but there is one action between them by which both approach each other."

Next Newton concluded that "according to this law all bodies must attract each other." He proudly presented the conclusion and explained why the magnitude of the attractive force is so small that it is unobservable. "It is possible," he wrote, "to observe these forces only in the huge bodies of the planets."

In book three of the *Principia,* which is also concerned with the system of the world but is somewhat more mathematical, Newton treats the topic of gravitation in essentially the same way. First, in what is called the moon test, he extends the weight force, or terrestrial gravity, to the moon and demonstrates that the force varies inversely with the square of the distance. Then he identifies the same terrestrial force with the force of the sun on the planets and the force of a planet on its satellites. All these forces he now calls gravity. With the aid of the third law of motion he transforms the concept of a solar force on the planets into the concept of a mutual force between the sun and the planets. Similarly, he transforms the concept of a planetary force on the satellites into the concept of a mutual force between planets and their satellites and between satellites. The final transformation is the notion that all bodies interact gravitationally.

My analysis of the stages of Newton's thinking should not be taken as diminishing the extraordinary force of his creative genius; rather, it should make that genius plausible. The analysis shows Newton's fecund way of thinking about physics, in which mathematics is applied to the external world as it is revealed by experiment and critical observation. This way of thinking, which I call the Newtonian style, is captured by the English title of Newton's great work: *Mathematical Principles of Natural Philosophy.*

The Newtonian style consists in a repeated give-and-take between a mathematical construct and physical reality. In the development of Newton's ideas on gravity and in his presentation of those ideas in the *Principia,* he started with a mathematical construct that represents nature simplified: a point mass moving around a center of force. Because he did not assume that the construct was an exact representation of the physical universe he was free to explore the properties and effects of a mathematical attractive force even though he found the concept of a grasping force "acting at a distance" to be abhorrent and not admissible in the realm of good physics. Next he compared the consequences of his mathematical construct with the observed principles and laws of the external world such as Kepler's law of areas and law of elliptical orbits. Where the mathematical construct fell short Newton modified it. He made the center of force not a mathematical entity but a point mass. I say a point mass rather than a physical body because he had not yet considered physical properties such as size, shape and mass.

From the modified mathematical construct Newton concluded that a set of point masses circling the central point mass attract one another and perturb one another's orbits. Again he compared the construct with the physical world. Of all the planets, Jupiter and Saturn are the most massive, and so he sought orbital perturbations in their motions. With the help of John Flamsteed, Newton found that the orbital motion of Saturn is perturbed when the two planets are closest together. The process of repeatedly comparing the mathematical construct with reality and then suitably modifying it led eventually to the treatment of the planets as physical bodies with definite shapes and sizes.

After Newton had modified the construct many times he applied it to the system of the world. He asserted that the force of attraction, which he had derived mathematically, is universal gravity. He found that the moon moves as if it were attracted to the earth with a force that is 1/3,600th of the strength of the gravitational force with which the earth pulls on objects at its surface. Since the moon is 60 times farther from the center of the earth than objects on the earth's surface are, the factor of 1/3,600 is consistent with the deduction that the earth's gravity extends to the moon and diminishes with the square of the distance.

The law of universal gravitation explains why the planets follow Kepler's laws approximately and why they depart from the laws in the way they do. It demonstrates why (in the absence of friction) all bodies fall at the same rate at any given place on the earth and why the rate varies with elevation and latitude. The law of gravitation also explains the regular and irregular motions of the moon, provides a physical basis for understanding and predicting tidal phenomena and shows how the earth's rate of precession, which had long been observed but not explained, is the effect of the moon's pulling on the earth's equatorial bulge. Since the mathematical force of attraction works well in explaining and predicting the observed phenomena of the world, Newton decided that the force must "truly exist" even though the received philosophy to which he adhered did not and could not allow such a force to be part of a system of nature. And so he called for an inquiry into how the effects of universal gravity might arise.

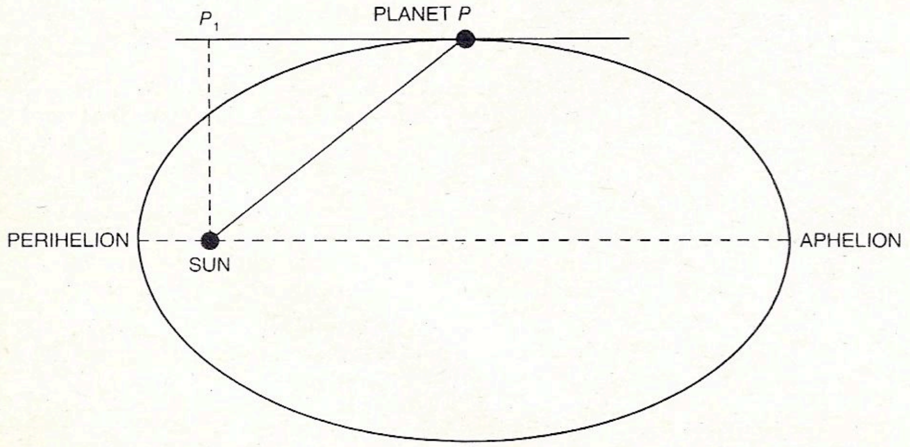
Although Newton at times thought universal gravity might be caused by the impulses of a stream of ether particles bombarding an object or by variations in an all-pervading ether, he did not advance either of these notions in the *Principia* because, as he said, he would "not feign hypotheses" as physical explanations. The Newtonian style had led him to a mathematical concept of universal force, and that style led him to apply his mathematical result to the physical world even though it was not the kind of force in which he could believe.

Some of Newton's contemporaries were so troubled by the idea of an attractive force acting at a distance that they could not begin to explore its properties, and they found it difficult to accept the Newtonian physics. They could not go along with Newton when he said he had not been able to explain how gravity works but that "it is enough that gravity really exists and suffices to explain the phenomena of the heavens and the tides." Those who accepted the Newtonian style fleshed out the law of universal gravity, showed how it explained many other physical phenomena and demanded that an explanation be sought of how such a force could be transmitted over vast distances through apparently empty space. The Newtonian style enabled Newton to study universal gravity without premature inhibitions that would have blocked his great discovery. The 18th-century biologist Georges Louis Leclerc de Buffon once wrote that a man's style cannot be distinguished from the man himself. In the case of Newton his greatest discovery cannot be separated from his style.

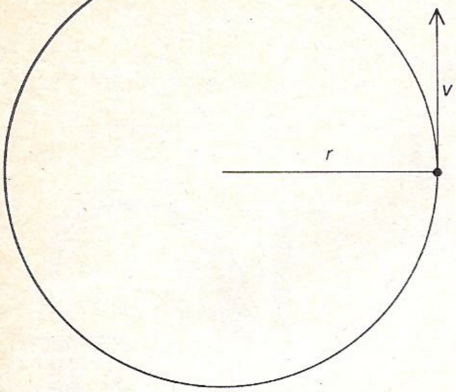
The correspondence between Hooke and Newton clearly shows that Hooke taught Newton how to analyze curved motion. Hooke subsequently made the much stronger claim that he deserved credit for suggesting to Newton the law of universal gravity varying inversely with the square of the distance. Many historians have echoed Hooke's view.

The claim, however, does not hold up. Hooke had merely suggested that the planets are subject to an inverse-square force directed toward the sun. Universal gravitation is much more than a solar-directed force. It also implies an effect of the planets on the sun. What is more, it applies to all objects in the universe. The law of universal gravitation is not merely an inverse-square relation; it is also a mathematical relation between the masses of the attracting bodies. It took tremendous insight to leap from an inverse-square solar-directed force to universal gravitation. And it took the genius of Newton to invent the modern concept of mass.

Newton did not feel he owed Hooke credit even for suggesting that the centripetal force is inversely proportional to the square of the distance. In 1673 Huygens had published a supplement to a book on the pendulum clock in which he states that for circular motion the centrifugal force is measured by v^2/r, where v is the velocity of the orbiting body and r is the radius of rotation. Newton had independently discovered the same relation in the 1660's. Since the mathematical difference between a centrifugal force and a centripetal force is only a matter of direction, the v^2/r relation also holds for a centripetal force. From this relation and Kepler's third law it follows by simple algebra that the centripetal force varies inversely with



ORBITAL SPEED OF A PLANET is inversely proportional not to the direct distance between the sun and the planet but to the perpendicular distance (the distance represented by the broken line between the sun and the tangent to the orbit *PP'*). Only at two points in the orbit (perihelion and aphelion) are the direct distance and the perpendicular distance the same.



$$\frac{v^2}{r} = \left(\frac{2\pi r}{T}\right)^2 \Big/ r = \left(\frac{4\pi^2 r^2}{T^2}\right)\frac{1}{r} =$$

$$4\pi^2\left(\frac{r^2}{T^2}\right)\frac{1}{r} = 4\pi^2\left(\frac{r^2 r}{T^2 r}\right)\frac{1}{r} =$$

$$4\pi^2\left(\frac{r^3}{T^2}\right)\frac{1}{r^2} = 4\pi^2 K\left(\frac{1}{r^2}\right)$$

INVERSE-SQUARE NATURE OF CENTRIPETAL FORCE for circular orbits can be deduced from Kepler's third law of planetary motion and from the law of centripetal force. According to Kepler's third law, r^3/T^2 is a constant K, where r is the radius of the planet's orbit and T is the period of the orbit. The law of centripetal force states that for a circular orbit the centripetal force is v^2/r, where v is the planet's velocity. In time T the planet makes a complete orbit, moving a distance $2\pi r$ (the circumference of a circle), and so the velocity is $2\pi r/T$.

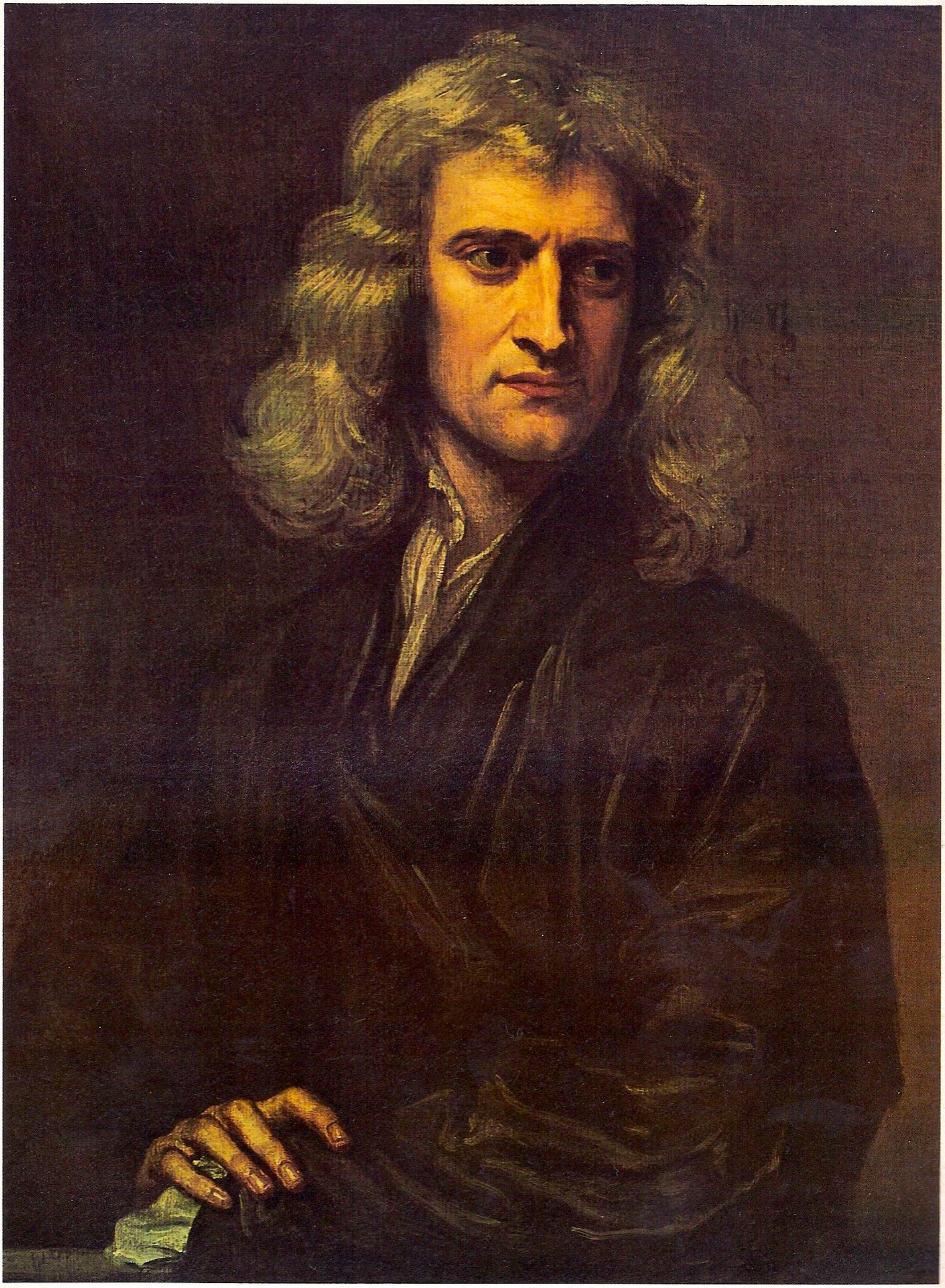
the square of the distance. After Huygens' book was published anyone with a rudimentary knowledge of algebra could have found an inverse-square centripetal force for a circular orbit. Accordingly Newton saw no need to acknowledge Hooke's statement of an inverse-square law.

Both Hooke and Newton were aware that finding an inverse-square law for circular orbits was not the same thing as showing that the law holds for elliptical orbits in which the motion follows Kepler's law of areas. The task, which Newton carried out, was to demonstrate that an inverse-square law of centripetal force corresponds to orbital motion according to Kepler's law of elliptical orbits and his law of areas. In discussing this point in the letter dated January 6, 1680, Hooke made a fundamental error that must have convinced Newton that Hooke did not entirely understand what he was talking about. Hooke said that if the attraction varies inversely as the square of the distance, the orbital speed of a planet will be "as Kepler supposes Reciprocall to the Distance." Yet under the conditions Hooke assumed the orbital speed is not inversely proportional to the direct distance from the sun except at the extreme points of the orbit: perihelion and aphelion. In view of Hooke's error Newton was not about to give him credit for having suggested the inverse-square nature of the centripetal force.

In 1717 Newton wanted to ensure his own priority in discovering the inverse-square law of gravitation, and so he invented a scenario in which he made the famous moon test not while writing the *Principia* but two decades earlier in the 1660's. The documents of the 1660's, however, indicate that he was not then comparing the falling of the moon in its orbit with the falling of objects on the earth but was comparing the "centrifugal endeavor" of the orbiting moon with the "centrifugal endeavor" of a body on the earth's surface rotating along with the daily motion of the earth. He did calculate that for circular planetary orbits the "centrifugal endeavor" would be inversely proportional to the planet's distance from the sun, but he drew no physical conclusions from the calculation.

Newton never published his invented scenario of the early moon test. He included it in the manuscript draft of a letter to the French writer Pierre Des Maizeaux but then crossed it out. Newton also circulated the familiar story that a falling apple set him on a chain of reflections that led to the discovery of universal gravitation. Presumably this invention was also part of his campaign to push back the discovery of gravity, or at least the roots of the discovery, to a time 20 years before the *Principia.*

The real roots of the discovery cannot be put any earlier than December, 1684, when Newton first recognized that if the sun attracts the earth, the earth must attract the sun with a force of equal magnitude. In 1685 he overcame his usual reluctance to write up his discoveries and started to draft the *Principia* for publication by the Royal Society. Perhaps his willingness to present his work for public inspection (and thereby risk possible disapprobation) was motivated first by his momentous discovery of interplanetary perturbations followed by his bold conception of universal gravity. He had within his grasp the foundation of a new system of natural philosophy that could be expounded on mathematical principles. In short, once Newton had something of real consequence to say about celestial dynamics he was willing and even eager to present it to the world.



PORTRAIT OF ISAAC NEWTON was painted by Godfrey Kneller in 1689, when Newton was 46. Four years earlier Newton had developed the concept of universal gravitation. Newton's principal work *Philosophiae Naturalis Principia Mathematica* was published in 1687.

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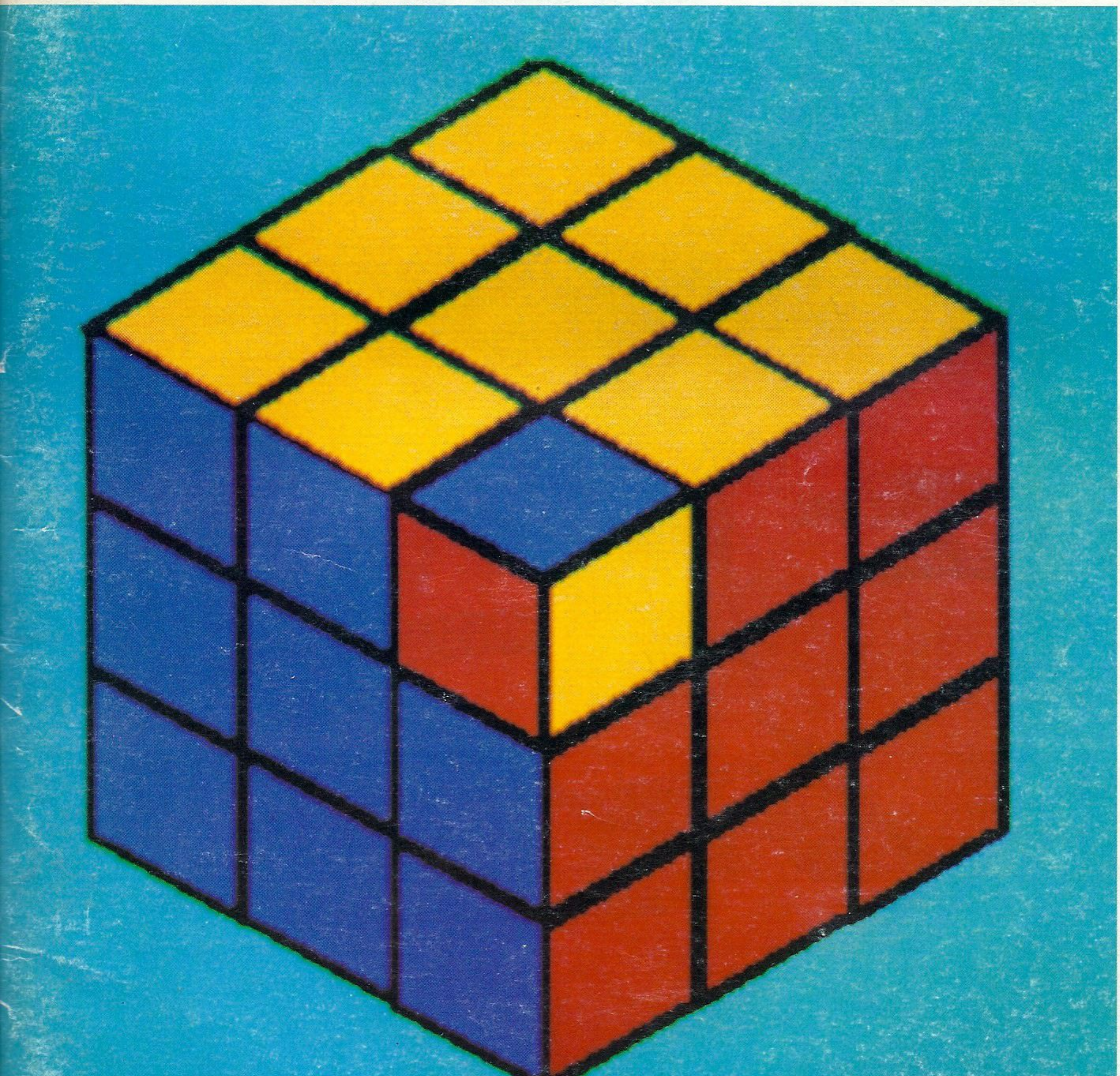

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