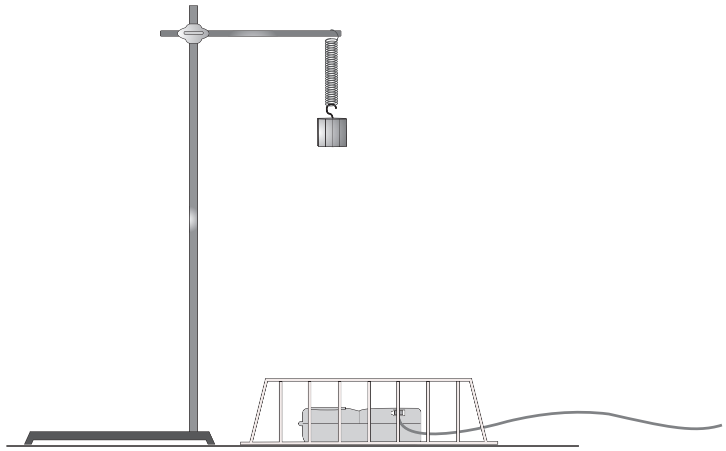
PART A–MATHEMATICAL MODEL FOR SIMPLE HARMONIC MOTION



The objective of this exercise is to compare the data for the position and acceleration of a spring-mass oscillator as a function of time (as found using a Motion Sensor/Low-g accelerometer set-up) with the mathematical model for such a system as previously derived in class ($y(t) = (v_0/\omega)\sin(\omega t) + y_0\cos(\omega t)$). The "fit" between the mathematical model and the actual data will be done by trial-and-error. By adjusting the parameters for y_0, v_0 and ω (which is accomplished by adjusting the spring constant k as $\omega = (k/m)^{1/2}$), a satisfactory fit can be achieved. Once this is done, one can calculate the velocity of the oscillator by taking the "numeric derivative" of the y vs. t data ($v = \Delta y/\Delta t$ where the acceleration is effectively constant for small values of Δt) and compare these velocities (and the accelerations found using the low-g accelerometer) to those expected using the mathematical model derived in class ($v(t) = dy/dt$ and $a(t) = d^2y/dt^2$).

The "raw" LoggerPro data (the position of the mass *relative to the motion sensor* every 1/30th of a second) can be found on the Excel spreadsheet that accompanies this document. Be sure to do the following:

- Find the <u>average</u> initial position of the spring-mass oscillator (*relative to the motion sensor*) when it is hanging at its **equilibrium position**.

- "Re-zero" the position data so that y(t) data represents the position of the mass *relative to the equilibrium position* ($y = 0$) during the course of the oscillation.

- "Re-zero" your time data so that your value for y_0 corresponds to $t_0 = 0$.

- Calculate (by "half-interval" method) your values for the velocity such that

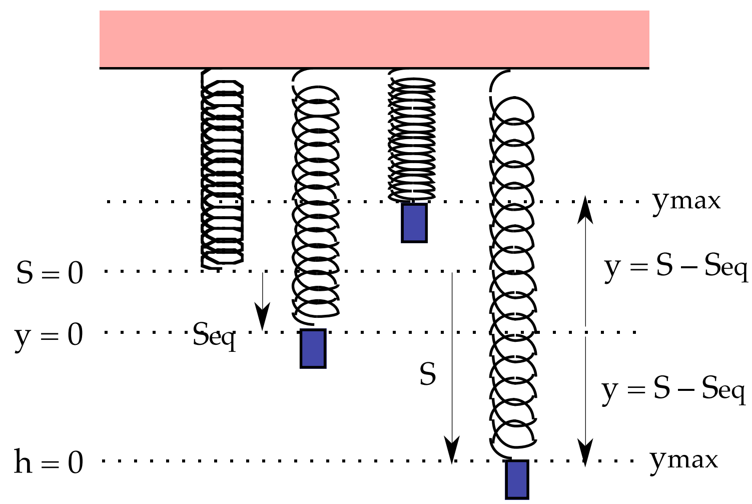
$$v_n = \frac{(y_{n+1} - y_{n-1})}{(t_{n+1} - t_{n-1})}$$

- Use the initial data point as the value for y_0. Set "reasonable" initial values for v_0 and k (to be adjusted by trial-and-error later using "sliders" for v_0, and k) and plot y_{expt} and y_{model} as a function of time on same plot. Remember to plot the "data points" (y_{expt}) as true data <u>points</u> and the values from the model ($y_{model} = (v_0/\omega)\sin(\omega t) + y_0\cos(\omega t)$) as a <u>smooth curve</u>.

- Adjust, by trial-and-error (using scrollbars ["sliders"]), the parameter values for k, and v_0 until you achieve a "good fit" between the experimental data and the model. [HINT: Set your values for y_0, v_0 first and then change k (and hence the period of the oscillation) accordingly].

- Plot v_{expt}, a_{expt}, v_{model} and a_{model} in a manner similar to that for y_{expt} and y_{model}. Note any differences between the experimental data and the mathematical models and account for these differences.

Part B–Energies of a Vertical Spring–Mass Oscillator



Once you've shown that the mathematical relationships accurately model the real physical system (and have determined, by trial-and-error, the spring constant, k, for your spring):

- Use the *experimental* data (not the results from the model) in order to calculate and plot the KE(t), EPE(t), GPE(t) and TE(t) as a function of time. Pay special attention to how to calculate the EPE of the spring. Remember that the EPE depends upon the total <u>stretch or compression</u> in the spring, not just the position of the mass relative to the equilibrium position. [NOTE: For this plot, you may plot the data as a smooth curve as opposed to individual data points.]

- Use the *experimental* data to plot the KE, EPE, GPE and TE as a function of <u>velocity</u>.

- Compare these plots to the same plots produced using the values obtained using the *mathematical model*.

- Plot the *experimental* acceleration as a function of the *experimental* position (y). What trendline could you fit to this data? Using the LINEST function,what information about the oscillator can be derived from this trendline? Compare this result to other values previously found in this lab.

Purpose + Procedure — Mathematical Methods and Physical Models

In an embedded Word document (or series of embedded Word documents) *on a separate worksheet*, you are expected to include a thorough, step-by-step discussion/description of the physics, including a clear statement of how (and why) you derived/calculated all values. Clear, well-labeled diagrams should be included as part of that discussion. This is your opportunity to demonstrate your depth of insight and understanding and to discuss what you learned by completing this exercise. You should use the drawing tools in Microsoft Word or Excel to produce annotated diagrams.

Analysis of Plots + Questions, Answers and Annotations

As part of your analysis, you should:

(i) Account for any differences between the experimental data and the mathematical models (y_{expt} and y_{model}, v_{expt} and v_{model}, a_{expt} and a_{model}).

(ii) Discuss and explain what you observe about the plot of KE, EPE, GPE and TE as a function of time. Account for any trends or observations.

(iii) Describe and discuss the plot the KE, EPE, GPE and TE as a function of velocity.

(iv) Describe and discuss the plot of acceleration as a function of position.

In addition to the questions below, be sure to address other general questions (as appropriate) found in the document ***Description of XL Lab Content—Format+Grading*** **found on the website.** As with the procedure, this analysis should be on a separate worksheet ***as an embedded Word document.***

1. Determine the first time that the block achieves its maximum downward acceleration. What is the value of the acceleration?
2. What is the speed of the block after 0.75 sec? Which way is the block moving? Is it speeding up or slowing down? Explain how you know.
3. Graphically estimate the period of the spring-mass oscillator. Compare this result to the value derived using $T = 2\pi/\omega$.

REMEMBER: All XL labs have "required elements" of content and style. Be sure to refer to the ***Grading Rubric for Advanced Physics XL Labs***, ***Description of XL Lab Content—Format+Grading*** and the ***Reference and Style Guide for Microsoft Excel*** in order to acquaint yourself fully with these requirements.