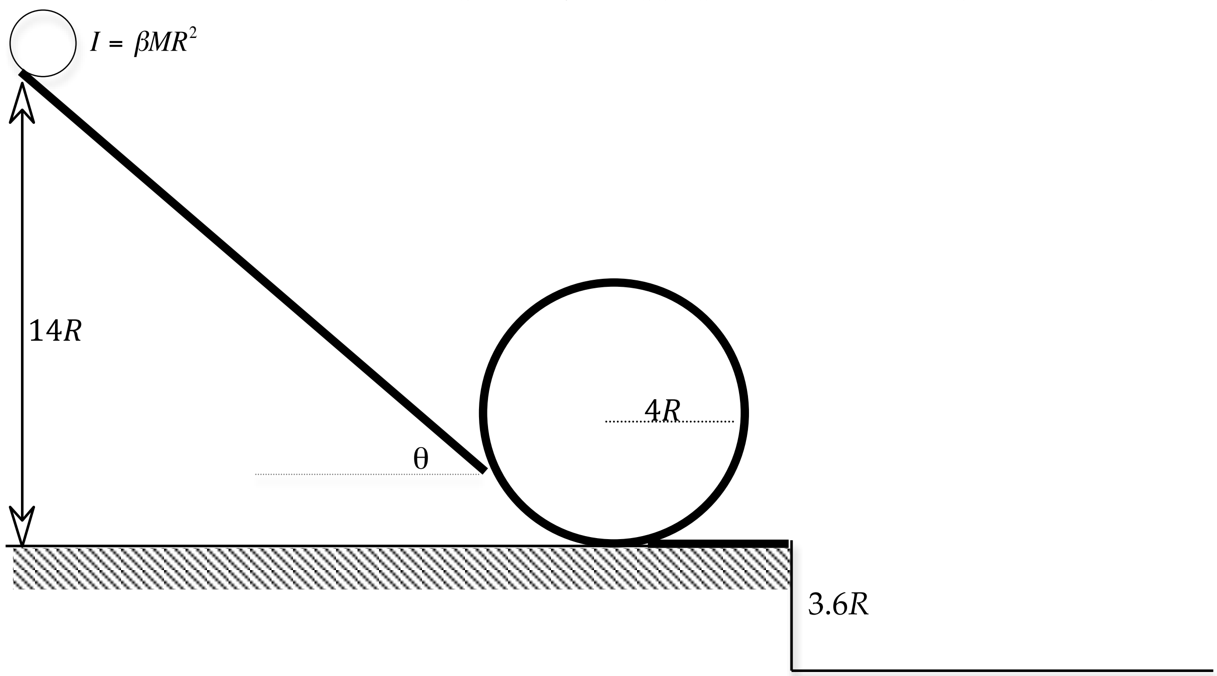
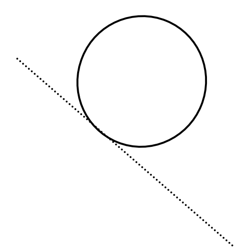
This problem set is open book and open notes only. You may not consult or confer with anyone *other than Mr. Burns*. Full credit will be awarded for each problem *only if* the correct answer is accompanied by sufficient work that is presented in such a manner that the logic and mathematical operations are clear and easy to follow. All answers must be boxed, labeled and accompanied by the appropriate units. Do each problem on a separate sheet.



1. A **solid** sphere of mass M, radius R, and rotational inertia, $I = \frac{2}{5}MR^2$, is placed on a ramp at a height of $14R$ above the platform. The ramp makes an angle θ with respect to a horizontal platform to which the ramp is affixed. The sphere is released from rest and *rolls without sliding* and enters a loop-the-loop with a radius of $4R$. The platform is a height of $3.6R$ above the floor. Express all algebraic answers in terms of given quantities and fundamental constants.



a. (2) On the diagram to the right, draw and label the fundamental forces acting on the sphere as it rolls down the ramp. Your vector arrows should begin at the point application of each force.

b. (4) Derive an expression for the acceleration of the center of mass of the sphere as it rolls down the ramp.

c. (5) Derive an expression for the normal force in the sphere when the sphere is *at the top* of the loop.

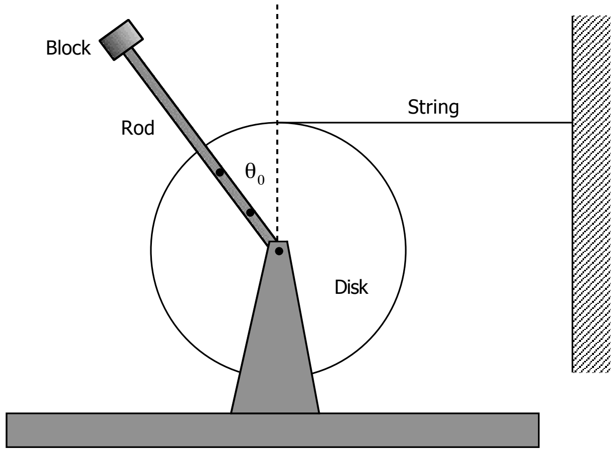
d. (6) Derive an expression for the horizontal distance from the edge of the platform to where the sphere lands on the floor after having completed the loop.

e. (3) Suppose that *the solid sphere is now replaced by a* ***hollow*** *sphere* ($I = \frac{2}{3}MR^2$) having the same mass, M, and radius, R. How will the distance from the edge of the platform to where the hollow sphere lands on the floor compare with the distance determined in part (d) for the solid sphere?

_______Less than _______The same as _______Greater than

Briefly justify your response.

2. As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk so that it will rotate with the disk. A block is attached to the end of the rod. The properties of the disk, rod and block are as follows:



Disk: mass = $3M$, radius = R, moment of inertia about center $I_D = (1/2)\ mR^2$

Rod: mass = M, length = $2R$, moment of inertia about one end $I_R = (1/3)\ mL^2$

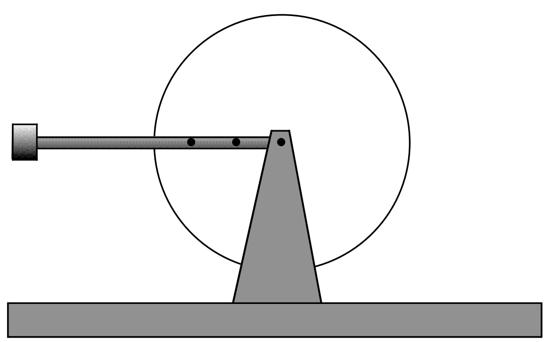
Block: mass = $2M$, $I_{PT.\ MASS} = mR^2$

The system is held in equilibrium with the rod at angle θ_0 to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Express your answers to the following in terms of M, R, θ_0, and g.

a. (5) Show that the tension in the string is $5Mg\sin\theta$.

The string is now cut, and the disk-rod-block system is free to rotate.

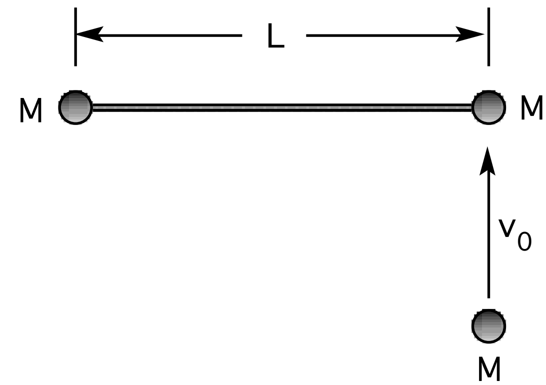
b. (5) Show that, at the instant after the string is cut, the magnitude of the angular acceleration of the disk is $\frac{6g\sin\theta}{13R}$.



As the disk rotates, the rod passes the horizontal position as shown above.

c. (8) Show that the linear speed of the mass located at the end of the rod (for the instant the rod is in the horizontal position) is $4\sqrt{\frac{3Rg\cos\theta}{13}}$.

3. (10) A space shuttle astronaut in circular orbit around the Earth has an assembly consisting of two small dense spheres (pt. masses) each of mass M, whose centers are connected by a rigid rod of length L, and negligible mass. The astronaut also has a device that will launch a small lump of clay of mass M at speed v_0. Express your answers in terms of M, v_0, L and fundamental constants.



Initially the assembly is "floating" freely at rest relative to the cabin. The astronaut launches the clay lump so that it perpendicularly strikes and sticks to one of the spheres of the assembly.

a. (2) Determine the distance **from the left end of the rod to the center of mass** of the system (assembly and clay lump) immediately after the collision. (Assume that the radii of the spheres and the clay lump are much smaller than the separation of the spheres.)

b. (1) On the figure above, indicate the direction of motion of the center of mass immediately after the collision. (Hint: Momentum is a vector quantity.)

c. (2) Determine the <u>linear</u> speed of the center of mass immediately after the collision.

d. (3) Determine the <u>angular</u> speed of the system (assembly and clay lump) **around (relative to) the center of mass** immediately after the collision.

e. (2) Determine the change in kinetic energy as a result of the collision.